Angular momentum of walking at different speeds

Bradford C. Bennett a,b,*, Shawn D. Russell b, Pradip Sheth b, Mark F. Abel a

a Department of Orthopaedic Surgery, University of Virginia, Kluge Children’s Rehabilitation Center, 2270 Ivy Rd., P.O. Box 800232, Charlottesville, VA 22903, USA
b Department of Mechanical and Aerospace Engineering, University of Virginia, Kluge Children’s Rehabilitation Center, 2270 Ivy Rd., P.O. Box 800232, Charlottesville, VA 22903, USA

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Abstract

Recently, researchers in robotics have used regulation of the angular momentum of body segments about the total body center of mass (CoM) to develop control strategies for bipedal gait. This work was spurred by reports finding that for a “large class” of human movement tasks, including standing, walking, and running the angular momentum is conserved about the CoM. However, there is little data presented to justify this position. This paper describes an analysis of 11 male adults walking overground at 0.7, 1.0, and 1.3 times their comfortable walking speed (CWS). The normalized angular momenta about the body CoM of 12 body segments were computed about all three coordinate axes. The normalized angular momenta were both small (<0.03) and highly regulated for all subjects and walking speed with extrema that negatively correlated with walking speeds. It was found that the angular momentum of the body about its CoM during walking could be described by a small number of principal components. For the adult walkers the first three principal components accounted for more than 97% of the variability of the angular momentum about each of the three principal axes at all walking speeds. In addition, it was found that the orthogonal principal components at each speed and for each subject were similar, i.e., the vectors of the principal components at each speed and for each subject were co-linear.

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* Corresponding author. Address: Department of Orthopaedic Surgery, University of Virginia, Kluge Children’s Rehabilitation Center, 2270 Ivy Rd., P.O. Box 800232, Charlottesville, VA 22903, USA. Tel.: +1 434 982 0513; fax: +1 434 982 1551.
E-mail address: bcb3a@virginia.edu (B.C. Bennett).
1. Introduction

Whereas the mechanics and control of walking are complex processes studied extensively, little attention has been dedicated to the understanding of angular momentum during locomotion. Recent work has suggested that angular momentum of the body's segments about its center of mass is highly controlled, can be approximated by a low dimensional descriptor, and could be a control parameter in walking (Herr & Popovic, 2008; Popovic & Englehart, 2004; Popovic, Hofmann, & Herr, 2004). Yet, there has been little data presented to validate this claim.

There has been little research on angular momentum during walking. Simoneau and Krebs (2000) performed a preliminary study of five elderly subjects and reported qualitative similarity of total body angular momenta curves between elderly fallers and non-fallers. Kaya, Krebs, and Riley (1998) also studied elderly subjects and found differences in the maximum HAT (head, arms, and torso) angular momenta in the sagittal and lateral planes between healthy elders and elders with bilateral vestibular hypofunction. Both studies found the whole-body angular momentum normalized by body mass was small (<0.03 m² rad/s).

Recently, Gu (2003) showed that whole-body angular momentum, relative to its center of mass, is highly regulated throughout the gait cycle in healthy adults (N = 5). Popovic et al. (2004) and Popovic and Englehart (2004) extended this work and found that for a single subject angular momentum primitives, obtained using principal component analysis (PCA), were invariant with walking speed.

The most complete analysis, to date, of angular momentum in walking was performed by Herr and Popovic (2008). They used a 16-segment model to describe the angular momentum of 10 individuals walking at their self-selected speed. Their earlier conclusions were confirmed since the total body angular momentum had a small absolute value and they found similarity between subjects. Thus they inferred that the angular momentum was highly regulated in walking. They also found that the first three principal components could explain more than 95% of the data along each of the three cardinal axes.

Here we examined the angular momenta of healthy adult males walking at three speeds; 0.7, 1.0, and 1.3 times their self-selected comfortable walking speed (CWS). We hypothesized that the normalized angular momenta about the CoM during walking would be small and have a similar pattern both at different speeds and for different subjects. We also hypothesized that the angular momenta could be described by low dimensional primitives. To test these hypotheses, we computed the height, mass, and velocity normalized three-dimensional angular momenta about the total body CoM and determined the degree to which segmental angular momenta cancelled one another. Moreover, we computed the principal components onto which segmental momenta were assembled and assessed the invariance of these components across subjects and walking speeds.

2. Methods

2.1. Subjects

Data were collected on 11 adult males with a mean age of 28.3 ± 12.4 years, a mean height of 181 ± 6.3 cm, and a mean mass of 76.5 ± 9.0 kg. The subjects were free of any neurological or musculoskeletal conditions or injuries (self-reported) that would have affected their gait. Subject consent was approved by the University of Virginia’s Human Investigation Committee and obtained for all subjects.

A full body Vicon Plug-in-Gait (PiG) set of 38 markers was attached to the subjects after anthropometric measurements were taken. Spherical markers (14 mm) were attached to the skin with double sided tape, with the exception of head and wrist markers which were attached to head and wrist bands, respectively. After a static trial the subjects’ CWS was determined from subjects walking continuously in the same oval loop used for testing. All kinematic data were taken on the middle 3 m of the 8 m side that traversed the motion capture system collection volume. In test trials subjects were guided by a pacer, set to the appropriate speed, that ran along the measurement side of the loop.

The subjects performed overground walking trials at three speeds (0.7, 1.0, and 1.3 times CWS), order randomized. Three-dimensional kinematic data were collected using a six camera Vicon Motion Analysis System at 120 Hz. At least 10 trials were performed. As is inherent in overground walking...
there was some variation in walking speed at each condition. The three strides that were closest to the desired speed with negligible acceleration and complete data were analyzed. This resulted in walking speeds within ±6.5% of the desired walking speed.

2.2. Subject specific computer model

A subject specific full body, 12-segment model, was created in Bodybuilder Language using VICON measurements and subject-specific anthropometric data (age, weight, height, and gender) using the method validated by Eames, Cosgrove, and Baker (1999). The 12 segments of the model were the head/neck, torso, upper arms (2), lower arms and hands (2), upper legs (2), lower legs (2), and feet (2). The center of mass (CoM) position of the total body was computed from the segment values. The segments’ kinematics were exported to Matlab code that computed the angular momenta.

2.3. Angular momentum

The angular momentum of a body is a vector quantity that represents the magnitude and the direction in which the body rotates about a reference point. The angular momentum of each segment was computed as the sum of the local angular momentum, the segment revolving about its own CoM, and a transfer term, the result of the CoM of the segment moving relative to the body CoM. These terms are defined for the ith segment as

\[ L_{\text{Local}} = I_{\text{CoM}} \omega_i \]

\[ L_{\text{Transfer}} = \bar{r}_i \times \bar{v}_i = \bar{r}_i \times m_i \bar{v}_i \]

where \( I_{\text{CoM}} \) is the moment of inertia tensor of the segment, \( \omega_i \) is the angular velocity vector, \( \bar{r}_i \) and \( \bar{v}_i \) are the relative position and velocity of the ith segment CoM to the whole-body CoM, respectively, \( \bar{p}_i \) is the linear momentum of the ith segment, \( m_i \) is the mass of the segment. For these computations the angular momentum of the body segments about the long axes of the segments was ignored as these terms were found to be very small. The total angular momentum is the sum of the momenta of all segments. The normalized angular momentum was created by dividing the angular momentum by the scalar quantities of the subject’s mass (kg), walking speed (m/s), and height (m) following the method of Herr and Popovic (2008).

The coordinate system used had the positive X axis pointing in the direction of walking, the positive Y-axis pointing up, and the Z axis in the medial lateral direction formed by the right hand rule. Thus the angular momenta in the X-direction reflect movements in the frontal plane, while moments in the Y- and Z-directions reflect movements in the transverse and sagittal planes, respectively.

2.4. Correlation coefficient

To compare the time dependent angular momentum curves between individuals the correlation coefficients were computed. The traditional correlation formulas for comparing two curves were extended to simultaneously compare the waveforms from all 11 subjects following the method of Gerstenfeld et al. (2003):

\[ \rho = \frac{\sum_{i=1}^{11} (X_i - \bar{X}) (Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{11} (X_i - \bar{X})^2 (Y_i - \bar{Y})^2}} \]

where \( X \) and \( Y \) are vectors of length \( n \) representing the two waveforms to be compared.

2.5. Principal component analysis

Principal component analysis (PCA) was used to explore whether the angular momentum data could be represented by a reduced data set. (An excellent description of PCA can be found in the article...
by Daffertshofer, Lamoth, Meijer, and Beek (2004).) PCA was performed on the time dependent angular momentum of each segment for each principal axis to obtain the set of 12 angular momentum primitives (the body was modeled with 12 segments) using standard Matlab functions. Since PCA basis vectors are data set dependent, the method of Krishnamoorthy, Goodman, Zatsiorsky, and Latash (2003) was applied to determine the similarity of basis vector sets. In this analysis a central vector of each set of vectors is determined and the dot product of a vector with the PCs in each plane is computed. The closer the dot product is to 1.0 the smaller the “angle” between the two vectors in 12-dimensional space.

2.6. Angular momentum cancellation

A measure was created to reflect the degree that the angular momenta of the body segments cancelled each other. The coefficient of cancellation, $\kappa$, was defined as

$$\kappa = \frac{\sum_{i=1}^{N} |L_i| - |\sum_{i=1}^{N} L_i|}{\sum_{i=1}^{N} |L_i|}$$

where $L_i$ is the angular momentum in a plane of the $i$th body segment and $N$ is the total number of segments. If there was no net angular momentum, all segments cancelled each other out perfectly, $\kappa = 1$. If there were no cancellation, then the two terms in the numerator are equal and $\kappa = 0$.

Data analysis was performed for each stride and these results analyzed. Repeated measures ANOVAs were performed on the dependent measures, including the mean coefficients of cancellation (3) and the percent of angular momentum variability accounted for by each principal component (9). Post-hoc analyses were performed with the Tukey honest significant difference test. Pearson correlations were used to determine velocity dependence of the data. Mauchly’s test was employed to test for sphericity of the data with the Greenhouse–Geisser correction applied to the univariate analyses. A Bonferroni correction was applied to the analysis and a statistical significance level of .004 was employed.

3. Results

Subjects walked at their CWS ± 0.3 CWS. This translated into walking speeds of 0.89 ± 0.12, 1.19 ± 0.21, and 1.57 ± 0.22 m/s for slow, CWS, and fast trials ($p < .0001$), respectively. To change speeds subjects varied both their stride and cadence. Stride length was 1.26 ± 0.12, 1.47 ± 0.20, and 1.72 ± 0.23 m for slow, CWS, and fast trials ($p < .0001$), respectively. Cadence was 0.71 ± 0.08, 0.81 ± 0.07, and 0.91 ± 0.07 strides/s for slow, CWS, and fast trials ($p < .0001$), respectively.

The normalized angular momenta were highly regulated in all trials. Correlations revealed that the patterns were similar between individuals with the correlation coefficients between .964 and .880 as shown in Table 1. The average momenta curves for all subjects at each speed are shown in Fig. 1 and are stereotypical of what has been reported in the literature (Herr & Popovic, 2008; Simoneau & Krebs, 2000). The momenta in the X-direction, frontal plane movements, reveal the small side to side motions of walking. The Y-direction total momentum, transverse plane movements, is smallest as the upper and lower body contributions are in opposite directions and cancel each other. The largest values of total body angular momenta tend to be from sagittal plane movements, Z-direction, and mostly are a product of the swinging legs in this plane. The extrema of the normalized momenta at different

Table 1
Correlation coefficients between subjects for total body normalized angular momentum for each plane at each speed.

<table>
<thead>
<tr>
<th>Plane</th>
<th>Slow</th>
<th>CWS</th>
<th>Fast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frontal</td>
<td>.9549</td>
<td>.9414</td>
<td>.9374</td>
</tr>
<tr>
<td>Transverse</td>
<td>.9637</td>
<td>.9652</td>
<td>.9363</td>
</tr>
<tr>
<td>Sagittal</td>
<td>.9142</td>
<td>.8517</td>
<td>.8803</td>
</tr>
</tbody>
</table>
speeds were different from each other \((p < .005)\) with a significant inverse correlation between the extrema values and walking speed as shown in Table 2. (Without velocity normalization the peak values increase with walking speed.) There was a tendency for the phase to shift with velocity, with the mean curves at the fast speed leading those at the slow speed by \(26^\circ\), \(22^\circ\), and \(9^\circ\) \((p < .02)\) in the transverse, sagittal, and frontal planes, respectively, reflecting the shifting in relative timing of the gait phases with walking speed.

### 3.1. Angular momentum cancellation

Plots of the cancellation coefficients for each plane at the different walking speeds are shown in Fig. 2. The total cancellations, areas under the curves, in the \(Y\) and \(Z\)-directions were found to be velocity dependent with cancellation increasing with increasing walking speed. The \(X\) momentum had the least cancellation with tendency of mean coefficient values to increase from .21 to .27 from

<table>
<thead>
<tr>
<th>Axis/plane</th>
<th>Max. angular momentum</th>
<th>Min. angular momentum</th>
<th>Velocity correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X)/frontal</td>
<td>Slow: 0.019</td>
<td>CWS: 0.016</td>
<td>Fast: 0.014</td>
</tr>
<tr>
<td>(Y)/transverse</td>
<td>Slow: 0.007</td>
<td>CWS: 0.006</td>
<td>Fast: 0.005</td>
</tr>
<tr>
<td>(Z)/sagittal</td>
<td>Slow: 0.025</td>
<td>CWS: 0.022</td>
<td>Fast: 0.018</td>
</tr>
</tbody>
</table>
slow to fast ($p < .01$). There was a phase shift in the $X$ momentum cancellation curves with the fast leading the slow data by 36° ($p < .0001$). There is little cancellation except in the middle of the swing/single support phases. The cancellation coefficient of the $Y$ momentum was larger with mean values that increased from .46 to .60 ($p < .0005$) from slow to fast, with peaks during double support. There was a tendency for the cancellation of fast walking to lead that of the slow trials by 11° ($p < .05$). The most cancellation was found in the $Z$ momentum with mean coefficients that increased from .70 to .79 ($p < .0005$) from slow to fast. There was nearly complete cancellation of the sagittal plane movements during swing/single support phases, but much less during double support when both legs move in the same direction relative to the CoM. The fast trial cancellation data tended to lead the slow trial cancellation data by 8° ($p < .01$).

### 3.2. Principal component analysis

The subject and velocity dependence of the principal components was examined using the method of central vector analysis (Krishnamoorthy et al., 2003). This analysis was performed on two sets of data. First, the PCs for each subject were computed and the central vector and dot products were computed. As shown in Table 3 the average dot products were greater than .96 and .87 for the first and second PCs, respectively. Secondly, the analysis was performed on the mean data at each walking speed and as shown in Table 3 the dot products were greater than .95 for the first and second PCs in all planes. Thus the PCs were similar between subjects and even more co-linear between speeds.

The mean PC coefficients for all trials are plotted for each body segment in Fig. 3. The coefficients reflect the relative contribution of each segment to angular momentum about a particular axis for a
particular principal component. Coefficients of opposite sign reveal segments whose angular momenta will tend to cancel each other. Examining the first PCs for each axis we see that for movements in the frontal plane the upper and lower body generate angular momenta in the same direction. In the transverse plane the upper and lower body generate angular momenta in opposite directions, which tend to cancel each other. In the sagittal plane, the right and left sides have opposite signs and we see most of the angular momentum is generated in the lower body.

Fig. 4 shows the time dependent PC scores or weighting coefficients on the mean momenta data for each axis for the first three PCs. The first PC has a period that is coincident with a single stride. The second PCs about the \( Y \)- and \( Z \)-axes have two periods coincident with a single stride.

The velocity dependence of the variability explained by each PC was also examined. In the frontal plane the percentage of data variance explained by each PC was independent of velocity (\( p = .30 \)), but this was not the case in the transverse and sagittal planes. In both planes the degree to which the data are explained by the first PC increased with increasing speed. In the transverse plane the percentage increased from 83.3% (slow) to 89.5% (fast) (\( p < .0001 \)), while in the sagittal plane the percentage increased from 82.0% (slow) to 88.1% (fast) (\( p < .0001 \)). The second PCs reduced from 7.6% to 6.1% (\( p < .05 \)) and 12.5% to 8.4% (\( p < .0001 \)) from slow to fast in the transverse and sagittal plane data, respectively. The third PCs went from 4.9% to 2.4% (\( p < 0.0001 \)) and 3.3% to 2.0% (\( p < .0005 \)) from slow to fast in the \( Y \)- and \( Z \)-data, respectively.

4. Discussion

Non-dimensional angular momentum patterns were found to be repeatable both between subjects and at different walking speeds, with an inverse relationship between the extrema of the angular momentum curves and walking speed. The source of this velocity dependent relationship appears to be an increase in cancellation of angular momenta with increased walking speed reflecting in part the relative shortening of the time spent in stance (Nilsson, Thorstensson, & Halbertsma, 1985; Stoquart, Detrembleur, & Lejeune, 2008), especially double support (Bejek, Paroczai, Illyes, & Kiss, 2006).

The effect of velocity on the angular momenta reveals a dynamic view of walking that provides a window into how our gait changes with walking speed. The decrease in the \( X \) momentum extrema at higher walking speeds reflects that there were lower side to side relative velocities at higher walking speeds. The decrease in the \( Y \) momentum with increasing speed may reflect in part a decrease in movement variability (Jordan, Challis, & Newell, 2007) and increase in the coherence (Donker, Daffertshofer, & Beek, 2005) between limbs. The increased coherence is reflected in the increase in momentum cancellation (from 46% to 62% from slow to fast walking) between the upper and lower body in the \( Y \)-direction. While the above argument can also be applied to the momenta in the \( Z \)-direction, we also see that the decrease in normalized angular momentum paralleled the decreasing amount of the gait cycle spent in double support. (This was likely impacting the data in the \( Y \)- and \( X \)-directions as well.) The less time spent in double support the longer relative time for the leg to swing and thus a slower leg velocity relative to the CoM. The longer relative swing time also increased the percentage of the gait cycle when cancellation of momenta was high, i.e., the two legs are generating opposing momenta. The changing time in double support was also reflected in the phase shifts of the momenta and cancellation curves with changing speed.
Fig. 3. The coefficients of the first three angular momentum principal components in the X-direction/frontal plane (top), Y-direction/transverse plane (middle), and Z-direction/sagittal plane (lower). The abscissa numbers correspond to the following segments: left (L) foot (1), right (R) foot (2), L shank (3), R shank (4), L thigh (5), R thigh (6), L upper arm (7), R upper arm (8), L forearm/hand (9), R forearm/hand (10), head/neck (11), torso (12). Values are the means and standard deviations for all subjects over all speeds.
4.1. Principal components

Principal component analysis revealed that the time dependent angular momenta could be represented by a reduced set of primitives. The first three principal components of the average data account for more than 97% of the data variation in all three planes. Looking at just the first two PCs, 88%, 94%, and 96% of the variability of the data is accounted for.

The analyses revealed that there was both variance and invariance with walking speed in the principal components. The PC vectors remained invariant, but the percentage of variance explained by the first PC increased with increasing speed with a corresponding decrease in the percent variance explained by the second and third PCs. The invariance of the PC vectors means that the directions (in the 12 dimensional segment angular momentum space) that had the most angular momentum variation were independent of walking speed. However the percentage of the data variation that was explained by each PC did change with velocity reflecting changes in the gait structure, such as the changing relative time spent in stance with changing walking speed, as noted above.

The effect of shorter stance times can be seen in the structure of the PCs and their weighting coefficients. As shown in Fig. 4, the weighting coefficients of the first PCs about the Y- and Z-axes were minimal (near zero) during double support (approximately 0–20% and 50–70% of the gait cycle) while the coefficients of the second PCs reach their maximums. During single support the situation reversed and the weighting coefficients of the first PCs about the Y- and Z-axes achieve their extremas while the second and third PCs reach their minimums (zero). Thus a shorter double support period increases the ‘influence’ of the first PC and decreases the contributions of the second and third PCs. In addition, the first PCs of Z and Y are responsible for most of the cancellation, both because they are the major contributors to the momentum and have components (coefficients) that generate momenta in different directions (are of opposite signs) as shown in Fig. 3. Thus while the structure of the PCs (the coefficients) did not change with velocity, the changes in the structure of the gait with increasing velocity resulted in a decrease in the total normalized angular momenta from the increased cancellation without any change in the structure of the PCs.

**Fig. 4.** Average normalized weighting coefficients for the X-direction/frontal plane (top), Y-direction/transverse plane (middle), and Z-direction/sagittal plane (lower). The solid blue lines are the first PC, the dotted lines on the second PC and the thick grey lines are the third PC in each plot. The gait cycle was defined as starting and ending with left heel contact (0% and 100%) with the right heel strike at 50%. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
4.2. Relationship of principal component coefficients and weightings to momentum cancellation

The coefficients of the PCs and their respective weighting coefficients provide insight into the generation and cancellation of angular momentum. As mentioned above only 21–27% of the X-direction total angular momentum was cancelled. This low value is suggested by the coefficients of the first PC all having the same sign. The second PC coefficients of the right and left sides of the body had opposite signs showing they tended to cancel each other, see Fig. 3, but this PC explained less than 11% of the dataset variability. The normalized weighting coefficients reveal the time dependent contributions of the PCs. The weighting coefficient for the first PC revealed the dominant contributions were from movements that repeat once each stride. We also see that at the points where the cancellation peaks, the weighting coefficient of the second PC was largest while the weighting coefficient of the first PC was near zero. The third PC explained only slightly less of the data, 10%, than the second PC and was almost 180° out of phase with that of the second PC.

In the Y-direction the coefficients of the first PC of the upper and lower body had opposite signs, see Fig. 3, reflecting our contralateral gait. They had their largest contributions during leg swing/single support. In the second PC the sides of the body opposed each other, see Fig. 3, and were most dominant during double support, see Fig. 4, creating the peaks in cancellation shown in Fig. 2. For the third PC the main contributors were the feet opposing the thighs, but this accounted for only 3% of the data.

The first PC coefficients, see Fig. 3, in the Z-direction revealed that most of the angular momentum was generated by the lower limbs and that the left and right sides opposed each other. This contribution was most dominant during swing phase where the legs create moments that are of opposite sign. During double support there was very little cancellation because both legs were moving in the same direction relative to the CoM. The second PC had the feet and shanks opposed by the thighs and head and torso momenta. This component, 10%, had its strongest effect during stance when there was little contribution from the first PC, see Fig. 4. The third PC was also dominated by the lower body with the two sides opposing each other, but explained only 2% of the data.

4.3. Limitations

The data presented here was computed with a 12-segment model. Thus some of our segments (e.g., forearm and hand) are combinations of actual body segments. Herr and Popovic (2008) presented data from a 16-segment model at their subjects’ self-selected walking speeds. The results at subjects’ self-selected speed presented here are very much in agreement with the findings of Herr and Popovic. Their time dependent angular momenta and the principal components are qualitatively similar to those presented here. Our percent data explained by the first principal components was generally within a standard deviation of the Herr and Popovic data. The similarity of our results suggests that little information is lost in going to a 12-segment model. In addition, we have made a detailed comparison of the results with our 12-segment model to 17-segment model results and found no lost of information at the total body level.

4.4. Implications for motor control

It appears the angular momenta are both highly controlled and well organized. The angular momenta peaks and cancellation were velocity dependent, as was the percentage of variation explained by the PC vectors reflecting changes in the sagittal and transverse planes. However, the organization of the angular momenta of the body segments, the PC coefficients, was independent of walking speed. In addition, this organization can be represented by only two or three primitives per plane as opposed to the 12 body segments used to model the body. This organization is highly suggestive of a neuromuscular synergy that can in part be scaled by velocity. Further work is underway applying the uncontrolled manifold hypothesis (UCM) of Scholz and Schöner (1999) to explore angular momenta conforms to this definition of synergy.
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